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# A Numerical SC Approach for a Teleoperated 7-DOF Manipulator

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## Abstract

*To tackle the singularity problem, the SC (Singularity-Consistent) approach was introduced. It achieves very stable control at and around a singularity with feasible joint velocities and no directional error in the end-effector velocity. This approach is very suited for a direct manual teleoperation system because of its error-less character for the direction. Until now, it was applied for the teleoperation of a non-redundant manipulator. In this paper, the SC approach for a 7-DOF manipulator will be addressed. To derive the whole properties of the SC approach, analytical studies for both the adjoint and the determinant of the Jacobian are necessary. However, it is very difficult to realize this goal, since the kinematics of a 7-DOF manipulator is quit complicated. Therefore, here, we establish a method to apply the SC approach to a 7-DOF manipulator numerically without analyzing the kinematic properties. This method, though, cannot realize the whole properties of the SC approach but a stable control at and around the singularities is achieved which is the most important and demanded property of the SC approach. Moreover, this method can also be applied for any type of articulated manipulator if its Jacobian can be defined. The results of our approach have been confirmed by experiments with graphics model.*


## 1. Introduction

During teleoperation, the operator is liable to move the slave arm to or around a singularity without perception. It is very dangerous since large joint velocities would occur or the system will become uncontrollable. Therefore the workspace of the manipulator is to be restricted not to allow it to reach the singularities practically. But, this method spoils a large part of the workspace. To tackle this problem, the DLS (Dumped Least Square) method was introduced [1], [2], [3]. The DLS method realizes stable motion around the singularities, but the end-effector deviates from the commanded direction. It means that the manipulator gets out of control for a while when it approaches a singularity. This phenomenon is not acceptable especially for the direct manual teleoperation. To tackle this problem, recently, the SC (Singularity Consistent) approach was introduced [4], [5], [6]. This approach can realize the reference direction exactly along with feasible joint velocities at and around the singularities. But, the end-effector cannot follow the exact commanded magnitude of the reference velocity. However, the directional error-less property is very suited for the manual teleoperation systems. Until now, this approach was applied to teleoperate the non-redundant 6-DOF manipulators [7].

In general, the redundancy is quite helpful to realize the complex tasks. Therefore, recently, the demand for the redundant manipulators is much increasing. But, unfortunately, their kinematics is complicated. Especially the kinematics of the redundant manipulators with displacement of joint axes (non-zero joint offsets) to fold themselves compactly is very complicated. The Robotics Research K-1207 arm which was developed for space applications is one of the representative examples [9]. It is well known that the redundant motions can be used to avoid the singular configurations. However, in teleoperation, even the redundant motion should be under the operator's control, since the arbitrary motions would induce serious problems like collisions against obstacles, etc. The concept of the arm plane [10], [11] is one of the solutions to handle the redundant motions. However, the arm plane induces some additional algorithmic singularities. Until now, the DLS method has been used to handle both the algorithmic and the kinematic singularities in spite of having the tracking errors.

On the other hand, the SC approach requires the analyses of both the Jacobian and its determinant. But, the analytical kinematic solution of a 7-DOF manipulator is too complicated to analyze. Therefore in this article, we establish a numerical method to employ the SC approach to a teleoperated 7-DOF manipulator instead of analyzing its kinematics. Though, this method cannot realize all the salient features of the SC approach, yet a stable control at and around the singularities, which is the most demanded feature of the SC approach, is achieved. Furthermore, the proposed method can be applied to any type of articulated manipulator, if its Jacobian can be defined. The results of the proposed approach have been confirmed by experiments with a graphics model.

## 2. Fundamentals

### 2.1. SC approach

First of all, the fundamentals of the SC approach are addressed [7]. The inverse kinematics is written generally as:

$$\begin{aligned} \dot{\boldsymbol{\theta}} &= \boldsymbol{J}^{-1}\dot{\boldsymbol{x}} \\ &= \frac{1}{\det \boldsymbol{J}}(\mathrm{adj}\boldsymbol{J})\dot{\boldsymbol{x}}, \end{aligned} \tag{1}$$

where $\boldsymbol{J}$ is the manipulator's Jacobian, $\det \boldsymbol{J}$ and $\mathrm{adj}\boldsymbol{J}$ denote its determinant and adjoint, respectively. The cause of occurrence of a singularity is the determinant approaching to zero. Note, that the term $(\det \boldsymbol{J})^{-1}$ is a scaling factor that is related to the magnitude and the direction of the motion in the joint space. On the other hand, $(\mathrm{adj}\boldsymbol{J})\dot{\boldsymbol{x}}$ determines the velocities of the individual joints.

In order to overcome the singularity problem, following modification in Eq. (1) are introduced [5].

$$\dot{\boldsymbol{\theta}} = \sigma b\ (\mathrm{adj}\boldsymbol{J})\boldsymbol{u}_r \tag{2}$$

where $\boldsymbol{u}_r$ is the directional unit vector of the reference end-effector velocities, $\sigma$ is a sign variable ($\sigma = \pm 1$), and $b \geq 0$ is a scalar variable. With a proper design of both $\sigma$ and $b$, the manipulator can be controlled at singularities reducing the rank of Jacobian by one, and in its vicinity, without any error in the direction and with feasible joint velocities.

Here, we should note that there are two types of velocity relations at a kinematic singularity [6]

- *Type A:* $(\mathrm{adj}\boldsymbol{J})\boldsymbol{u}_r \neq \boldsymbol{0}$
- *Type B:* $(\mathrm{adj}\boldsymbol{J})\boldsymbol{u}_r = \boldsymbol{0}$.

In case of Type A relation, a special motion called *self-motion* is obtained. In this case, some of the joints are moving while the end-effector is motionless.

On the other hand, with Type B relation, all components of the vector $(\mathrm{adj}\boldsymbol{J})\boldsymbol{u}_r$ vanish, and motions would stop entirely. Analysis shows, however, that in many cases, it is the common factor of $(\mathrm{adj}\boldsymbol{J})\boldsymbol{u}_r$ and $\det\boldsymbol{J}$ that yields this phenomenon. Hence, we can obtain a proper solution by eliminating this common factor [8]. This motion is related with a through singularity motion and an on singularity motion. But, it is necessary to analyze both the determinant and the adjoint matrix of the Jacobian to find out the common factor. In addition, type B relation requires very accurate direction which should lie in the arm plane. Such a precise command direction cannot be realized without computer assistance. It means that the computer system should have a knowledge of such precise command directions previously.

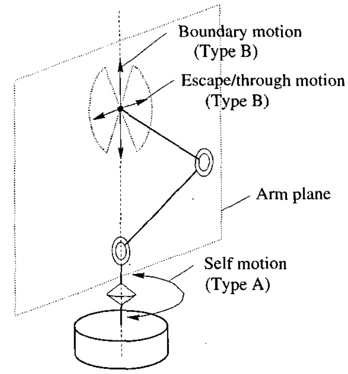


Figure 1. Type A and B relations at a shoulder singularity.

Note that the above two relations depend not only on the arm's configuration but also on the direction of the velocity command. An example of these relations are shown in Fig. 1.

### 2.2. Arm angle

In controlling a 7-DOF manipulator, the redundant motion needs to decided with some methods, e.g. minimum joint velocities [12], singularity avoidance [13], obstacle avoidance [14], etc. From the view point of teleoperation, the operator should be able to control all the motions always as he/she desires. In order to accomplish this target, the concept of arm angle was introduced [10], [11]. The arm angle is an angle between reference plane and arm plane which includes shoulder, elbow and wrist points. The definition of the arm angle $\psi$ is illustrated in Fig. 2, where S, E, W are placed on shoulder, elbow and wrist, respectively [9]. $\boldsymbol{v}$ is an arbitrary fixed unit vector (e.g. the unit vector in the vertical direction of the base plane). ED is normal to the projection of SE onto SW, and $\boldsymbol{l}$ is an orthogonal unit vector to SW in the reference plane that contains both $\boldsymbol{v}$ and SW. Consequently, the arm

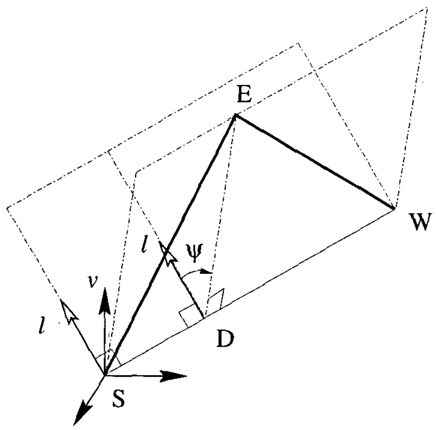



Figure 2. Illustration of the arm angle ($\psi$) concept.

angle $\psi$ is the angle between ED and $\boldsymbol{l}$.

This arm angle, however, introduces some additional algorithmic singularities. It means that the operator is required to handle both the algorithmic and the kinematic singularities simultaneously.

### 2.3. Augmented Jacobian

The differential of the relationship between the end-effector, arm angle and the joint angle can be written as:

$$\begin{pmatrix} \dot{\boldsymbol{x}} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}^{ee} \\ \boldsymbol{J}^{\psi} \end{pmatrix} \dot{\boldsymbol{\theta}} = \boldsymbol{J}^A \dot{\boldsymbol{\theta}} \tag{3}$$

where $\dot{\boldsymbol{x}} \in R^6$ is the end-effector velocity in the task space, $\dot{\psi} \in R^1$ is the arm angle's angular velocity, $\boldsymbol{J}^{ee} \in R^{6\times7}$ is the end-effector Jacobian, $\boldsymbol{J}^{\psi} \in R^{1\times7}$ is the arm angle's Jacobian, $\dot{\boldsymbol{\theta}} \in R^7$ is the joint velocity and $\boldsymbol{J}^A \in R^{7\times7}$ is the augmented Jacobian [15]. As a result, the problem of both algorithmic and kinematic singularities becomes a common singularity problem of a non-redundant system with 7 parameters.

## 3. SC approach for a 7-DOF manipulator

Here, we apply the SC approach to a 7-DOF manipulator having displacements of joint axes (non-zero joint offsets) to fold itself compactly. The Robotics Research K-1207 arm, developed for the space applications, is taken as a representative example. The kinematics of such a manipulator is quite complicated. Therefore it is very difficult to handle the type B singularities, since it requires to analyze both the determinant and the adjoint of the Jacobian. This fact leads us to employ the numerical methods to apply the SC approach to this 7-DOF manipulator. Unfortunately, such a method cannot handle the type B singularities. However, a stable control at and around the singularities can be achieved without directional errors and it is the most important feature of the SC approach.

### 3.1. Basic concept of the SC approach for a 7-DOF manipulator

To apply the SC approach to a 7-DOF manipulator, it is necessary to consider a proper scaling procedure such that the orientation, position and arm angle variables can be treated in a uniform manner. For this purpose, we introduce the constants $v_{max}$, $\omega_{max}$ and $\psi_{max}$ standing for the maximum translational, rotational and arm angle velocities, respectively. Using these constants, we can normalize the velocities, and obtain

$$\hat{\dot{\boldsymbol{p}}}_r = \hat{\boldsymbol{J}}^A \hat{\dot{\boldsymbol{\theta}}} \tag{4}$$

$$\hat{\dot{\boldsymbol{p}}}_r = \boldsymbol{T}_v^{-1} \dot{\boldsymbol{p}}_r \tag{5}$$

$$\hat{\boldsymbol{J}}^A = \boldsymbol{T}_v^{-1} \boldsymbol{J}^A \tag{6}$$

$$\begin{aligned} \boldsymbol{T}_v \;=\; & \sqrt{3}\mathrm{diag}[v_{max}, v_{max}, v_{max}, \\ & \omega_{max}, \omega_{max}, \omega_{max}, \psi_{max}] \end{aligned} \tag{7}$$

where $\dot{\boldsymbol{p}}_r$ is the reference velocity vector including $\dot{\boldsymbol{x}}_r$ and $\psi_r$, $\hat{\dot{\boldsymbol{p}}}_r$ is the normalized one of $\dot{\boldsymbol{p}}_r$, and $\hat{\boldsymbol{J}}^A$ is the normalized augmented Jacobian. The maximum of the norm of $\hat{\dot{\boldsymbol{p}}}_r$ would be equal to 1 due to the introduction of the factor $\sqrt{3}$ in the above equation. According to the SC approach, the end-effector velocity is represented as

$$\hat{\dot{\boldsymbol{p}}}_r = \nu_r \boldsymbol{u}_r \tag{8}$$

where $\nu_r$ is a scalar, and $\boldsymbol{u}_r$ is a unit vector. Hence we can obtain

$$\dot{\boldsymbol{\theta}} = \sigma \hat{b} \ (\mathrm{adj}\hat{\boldsymbol{J}}^A) \boldsymbol{u}_r \tag{9}$$

where $\hat{b}$ is a function of the manipulability and the end-effector velocity $\nu$ [5].

### 3.2. A numerical SC approach

As we mentioned, analytical solution of Eq. (9) is quite complicated. Therefore, a numerical approach is utilized. Of course we suppose that the Jacobian can be defined. Unfortunately, it takes a lot of computing time to obtain the adjoint matrix numerically. Therefore, the following modification

$$\begin{aligned} (-1)^n(\mathrm{adj}\hat{\boldsymbol{J}}_A)\hat{\boldsymbol{u}}_r = \boldsymbol{n}_H = [C_1, C_2, ...C_n]^T \\ C_p = (-1)^{p+1}\mathrm{det}\boldsymbol{H}_p \quad (p = 1, 2, ...n) \end{aligned} \tag{10}$$

is made, where $\boldsymbol{n}_H$ is the null space vector of a column augmented Jacobian $\boldsymbol{H}$ which is defined as

$$\boldsymbol{H} = [\hat{\boldsymbol{J}}_A, -\hat{\boldsymbol{u}}_r] \tag{11}$$

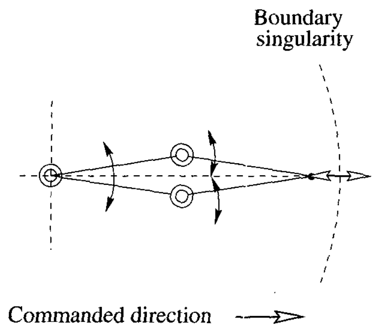



Figure 3. Chattering at the boundary singularity.

where $\boldsymbol{H}_p$ stands for the matrix $\boldsymbol{H}$ with $p$th column removed [5].

As a result, $(\mathrm{adj}\hat{\boldsymbol{J}}_A)\hat{\boldsymbol{u}}_r$ can be obtained from $\det\boldsymbol{H}_p$. It is easy to calculate the determinant of a matrix numerically by the well-known LU decomposition method.

### 3.3. Deciding the sign

One of the big advantages of the SC approach is its ability of reconfiguration using the Cartesian commands. To utilize this feature, the problem of chattering motions related to the decision of sign function $\sigma$ should be solved. Fig. 3 shows the chattering motions that are typical problem in teleoperation systems. While calculating the inverse of Jacobian conventionally, the determinant of the Jacobian includes the sign information into itself. However, if $\sigma$ is decided by the sign of this determinant, a chattering motion would occur at a boundary singularity, since the end-effector tries to keep up with the commanded direction. Of course this happens only in a discrete time system with digital servos. Therefore, maintaining the same sign while crossing through a singularity can be considered as a solution[5]. Fig. 4 illustrates this concept. $\sigma$ succeeds the previous sign until the command approaches zero. As a result, the end-effector even moves in the opposite direction for a while, but, it can realize a smooth reconfiguration. Unfortunately, in the case of a 7-DOF manipulator, the situation becomes more complex since the manipulator is apt to meet singularities continually. As a result, it is difficult to find an effective rule to decide about the sign using only the determinant of Jacobian.

In order to avoid this situation, we introduce a simple but very strong method. The chattering motion is caused by drastic alternations in the joint velocities. Therefore, the proposed method finds out the maximum among all the joint velocity drifts for each sign in each sampling interval. Only then, among these two maximum known drifts, the sign of the one with lesser magnitude is selected. As a result, very smooth motions can be realized easily. We also adopt a strategy that adjusts the sign according to the one of the determinant after passing through a zero command. We would like to emphasize that this method can be applied for any type of manipulator without having knowledge of its kinematics.

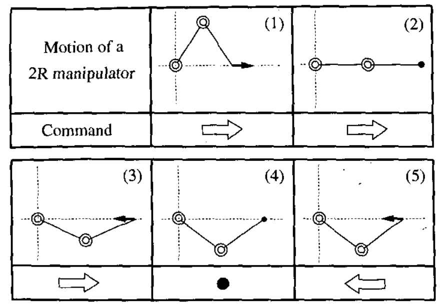



Figure 4. Motion through a singularity.

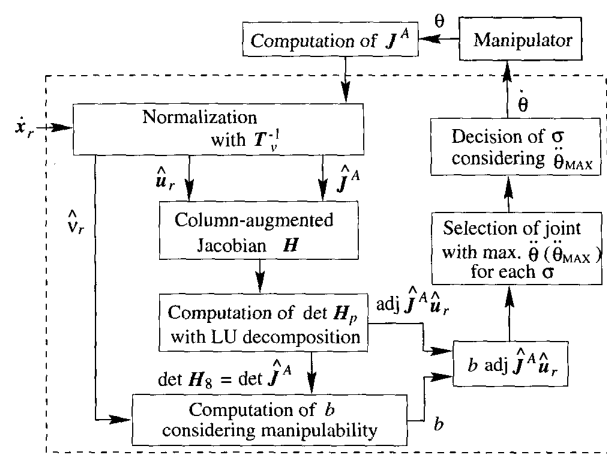



Figure 5. Algorithm for the numerical SC approach.

### 3.4. The numerical SC approach algorithm

Fig. 5 summarizes the above mentioned procedures in the form of an algorithm. What needs to be emphasized is that the computations inside the dotted box are purely numerical. It means that this method can be applied to any type of articulated manipulator without any need to analyze its kinematics, but only if its Jacobian can be defined.

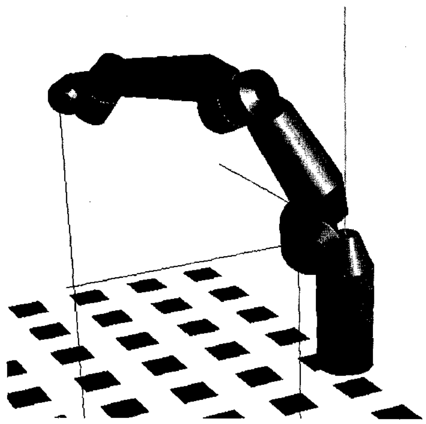


Figure 6. Computer graphics of the Robotics Research K-1207.

## 4. Experiments

We apply the proposed method to the graphical simulator of the Robotics Research K-1207. Fig. 6 shows an overview of the computer graphics of the manipulator. In this simulator, the operator can control the end-effector velocity of K-1207 using a 3D mouse that can generate the 6 axes commands simultaneously. However, the arm angle velocity is controlled by the keyboard. Fig. 7 shows an overview of the experimental setup. The DLS method is also applied to this manipulator to make a comparison with the proposed SC approach. One example experiment is shown in Fig. 8. A red (light) arrow and a blue (dark) arrow display the reference and the real end-effector velocities, respectively. It is easily notable that there is no directional error when the proposed SC approach is working. While in case of the DLS method, the manipulator even becomes uncontrollable for a while. In addition, we would like to emphasis that during self motions, there are no end-effector motions with the SC approach, but the end-effector does move while using the DLS method.

## 5. Conclusion

We established a numerical method to apply the SC approach to a teleoperated 7-DOF manipulator system. Though this method cannot really utilize all the properties of the original analytical SC approach, yet a stable control at and around the singularities, the most important character of the SC approach, is achieved without any directional errors. Furthermore, the proposed method can be applied to any type of articulated manipulator if its Jacobian can be defined. The validity of this approach has been confirmed by the experiments with a graphics model of a 7-DOF

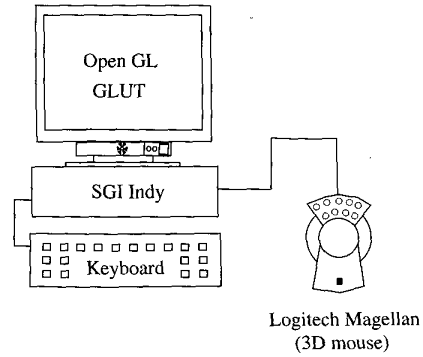
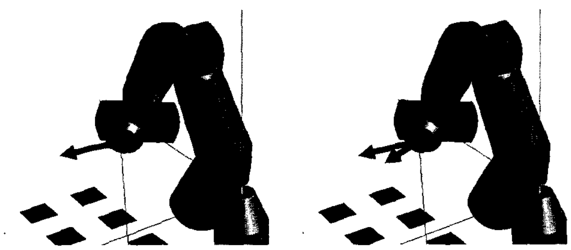



Figure 7. Experimental setup.



SC approach DLS method

Figure 8. Difference between the SC approach and the DLS method.

manipulator.

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